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NAMED AFTER MUHAMMAD AL-KHWARIZMI

MUHAMMAD AL-XORAZMIY NOMIDAGI
TOSHKENT AXBOROT TEXNOLOGIYALARI
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MATHEMATICAL MODEL FOR CONTROLLING A MECHATRONIC MODULE DISPLAYING 3D GRAPHICS IN ACCORDANCE WITH SPECIFIED COORDINATES AND POSITION

Rakhimov Temurbek Omonboyevich
Department of Mechatronics and Robotics, Faculty of Electronics and Automation Engineering, Tashkent State Technical University named after Islam Karimov, Tashkent, Uzbekistan
rahimov_timur@bk.ru

Fakhriddinov Azizbek Farkhod ugli
Department of Mechatronics and Robotics, Faculty of Electronics and Automation Engineering, Tashkent State Technical University named after Islam Karimov, Tashkent, Uzbekistan
faxriddinovazizbek042@gmail.com

Abstract. This article focuses on the development of a mathematical control model for a mechatronic module displaying 3D graphics based on specified coordinates and position. The paper examines the transformation of trajectory coordinates from a surface coordinate system to the reference coordinate system of the mechatronic module, as well as accounting for possible angular displacements (misalignments) in orientation. Furthermore, when working with an object in mechatronic modules displaying 3D graphics, a method for constructing actuator motion trajectories is proposed. This method divides the surface into elementary sections and develops mathematical models for them using equations for coupling between coordinate systems. When generating actuator motion trajectories, an algorithm for bypassing restricted areas is developed, based on optimization and motion minimization criteria, using a tree-like structure to determine the optimal sequence of movements. To ensure stable force interaction between the actuator and the surface, a two-level structural-parametric force control method is developed, taking into account the difference between actual and specified force values. The simulation results, based on the optimization and minimization criteria of movements, showed that the proposed mechatronic module control algorithm ensures increased positioning accuracy, reduced computational costs, and stable maintenance of specified pressing forces.

Keywords: *Mechatronic module for displaying 3D graphics, mathematical control model, determination of optimal motion sequence, optimization and minimization criterion.*

INTRODUCTION

When developing mathematical control models for a mechatronic module displaying 3D graphics, spatial visualization systems typically require high positioning accuracy and stability when reproducing 3D images on surfaces of various shapes and geometries. Therefore, the development of a mathematical control model for a mechatronic module that ensures the reproduction of three-dimensional graphic objects in accordance with specified coordinates and spatial position is particularly relevant [1]. Creating such a model requires solving a number of interrelated problems. These include converting trajectory coordinates from the surface coordinate system to the base coordinate system of the mechatronic module, predicting the motion of actuators given specified coordinates and positions, as well as permissible movement zones, and optimizing the speed of the working tool given the geometric features of the surface [2]. Furthermore, an important task is to ensure stable force interaction between the working tool and the surface by adjusting the pressing force, as this directly impacts display quality and positioning accuracy. The formation of the motion trajectory of the actuator of a mechatronic module displaying 3D graphics is based on dividing the overall trajectory into elementary sections, using their mathematical description and equations of the relationship between the coordinate systems of the surface and the module. A number of studies exist in this area, primarily examining issues of improving the static and dynamic characteristics of the mechatronic module [1,3]. However, the issues of improving and simultaneously taking into account its static and dynamic characteristics when developing a mathematical model of the control process remain insufficiently studied [4,5]. The proposed approach allows for the formation of a holistic mathematical model that takes into account the kinematic and dynamic properties of the mechatronic module during 3D visualization.

RESEARCH METHODOLOGY

In the case of precise installation and orientation of the module relative to the work surface, its coordinate system is parallel to the installation's base coordinate system:

$$x = L^* - x_p, y = s - y_p, z = z_p.$$

If the module's precise orientation is not maintained during installation, it is necessary to take into account the angles characterizing the misalignment of the module's coordinate systems and the base system. In this case, the module manually determines the coordinates of four surface boundary points, and the coordinates of three points at the module's base are also specified. Misalignment angles are determined using the scalar product of the corresponding vectors. The relationship between the surface and module coordinate systems is described by the following expressions:

$$\begin{aligned} x &= (L^* - x_p) \cdot \cos\phi_1 \cdot \cos\phi_3\,; \\ y &= (s - y_p) \cdot \sin\phi_1 \cdot \cos\phi_2\,; \\ z &= z_p \cdot \sin\phi_2 \cdot \sin\phi_3. \end{aligned} \qquad (1)$$

The research results showed that the developed method for constructing motion trajectories for the mechatronic module's actuator enables the implementation of rectilinear trajectories and circular arcs, taking into account the overall dimensions of the working surface, as well as the angles of misalignment of the module's orientation relative to the surface [4,6].

A specific feature of motion planning for the mechatronic module's actuator is the need to bypass exclusion zones. To solve this problem, a planning method was developed that ensures the bypass of exclusion zones, based on the use of the proposed mathematical description of typical trajectories, as well as data on the shape, dimensions, and location of exclusion zones [1,6]. The workpiece surface is represented as a set of sections

$$\Phi = \{\phi_1, \dots, \phi_r\},$$

where r is the number of sections. The components of this set are the trajectory sections that do not contain non-technological transitions:

$$\phi_r = \left\langle y_{i,j}^{(r)}, z_{i,j}^{(r)} \right\rangle,$$

where $y_{i,j}^{(r)}, z_{i,j}^{(r)}$ are the coordinates of the tool on the trajectory section in the coordinate system of the mechatronic module; $i = 1 \dots N_r,\ j = 1 \dots M_r$ are the numbers of the line and point on section r. The coordinates of the points on the trajectory are specified by functions whose form is determined by the algorithm for bypassing forbidden zones.

$$\begin{aligned} y_{i,j}^{(r)} &= f_y^{(r)}(i, j, h_f, v_f), \\ z_{i,j}^{(r)} &= f_z^{(r)}(i, j, s_s, s_f, z_f)\,. \end{aligned} \qquad (2)$$

where s_f, h_f are the width and height of the restricted area; y_f, z_f are the coordinates of the base point of the restricted area in the coordinate system of the mechatronic module.

The main condition for developing a forbidden zone traversal algorithm is minimizing the number of non-technological transitions $\Theta \rightarrow$ min. To solve this problem, the workpiece surface is divided into sections $r_j (j = 1 \dots n)$ (Fig. 1), and the sequence of their traversal is determined based on the coordinates of the last point of the previous section. Options for combining the simplest surface sections are represented as a tree, in which nodes represent sections indicating the direction of movement of the mechatronic module's actuator within these sections [6, 7]. Transitions between sections are depicted as solid lines with arrows in the absence of non-technological transitions and as dashed lines in their presence.

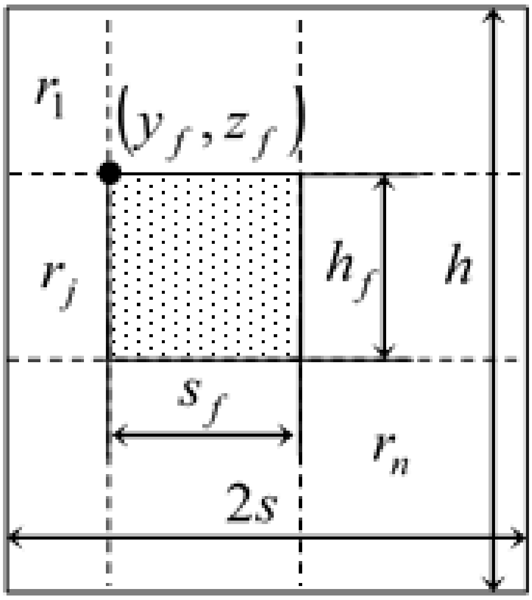


Fig. 1. By passing restricted areas during movement of the mechatronic module.

The tree is constructed according to a rule whereby surface areas processed without creating non-technological transitions are added to the left branch. Generally, the tree for traversing areas is described by a system of sets:

$$S = \{\Phi, P_t, P_{nt}, B_a, B_b, F\},$$

where: Φ is the set of surface areas; P_t, P_{nt} are the sets of technological and non-technological transitions between areas; B_a, B_b are the sets of input and output areas; F is the incidence ratio.

In this case, the following conditions are satisfied:

$$P_t \cap P_{nt} = \emptyset, B_a \cap B_b = \emptyset.$$

The incidence relation F indicates that for each transition there is a unique element $\langle B_a, P_t, B_b \rangle \in F$ or $\langle B_a, P_{nt}, B_b \rangle \in F$, which defines for it the input B_a and output set B_b of sections [1,8,9]. The minimum number of non-technological transitions is achieved if the number of equidistant lines in the first two sections is of the same parity, i.e. the condition is satisfied:

$$\lambda_1 \cdot \lambda_2 = 1,$$

$$\lambda_i = \begin{cases} -1, mod(h_y, 2a) \in (0, a] \\ 1, mod(h_y, 2a) \in (a, 2a]. \end{cases} \quad (3)$$

where λ_1, λ_2 are the parity of equidistant lines on the 1st and 2nd surface sections; h_y is the height of the surface section; a is the width of the equidistant line of the trajectory; $\mathrm{mod}(h_y, a)$ is the function for calculating the remainder from dividing h_y/a.

ANALYSIS AND RESULTS

The simulation results showed that the developed method effectively determines the optimal sequence for bypassing restricted areas by the mechatronic module's actuator, minimizing the number of non-technological transitions.

To ensure the required force interaction between the mechatronic module's actuator and the workpiece, a method for structural-parametric correction of pressing forces was developed. It is based on two-level force regulation depending on the magnitude of their misalignment. The actual force signal, read from the force-torque sensor, is compared with the following constraints:

$$\chi_1 = Nd_{i_{min}},\ \chi_2 = N_d(t_i) - N_{max}.$$

In the process of controlling the executive body of the mechatronic module, the requirements for permissible deviations of the trajectory coordinates and forces must be simultaneously met:

$$\max \mid \bar{q}_d - \bar{q}_0 \mid < \delta_q, \max \mid N_d - N_0 \mid < \delta_N,$$

where δ_q, δ_N are the permissible control errors for the generalized coordinates and for the pressing force; $\bar{q}_0, N_0$ and $\bar{q}_d, N_d$ are the specified and actual values of the generalized coordinates and forces at the contact point [10]. If, during movement along a given trajectory, the force at the contact point goes beyond the permissible limits, a correction of the moments in the degrees of freedom is carried out while maintaining the required accuracy of the trajectory processing:

$$\max \mid \bar{q}_0 - \bar{q}_0 \mid < \delta_q.$$

This is accomplished by calculating at each step of the control vector the correction values of the generalized coordinates [11]. If the trajectory of the actuator's movement is measured with a relatively small step, it is sufficient to correct the coordinates at the next *i*-th point (Fig. 2).

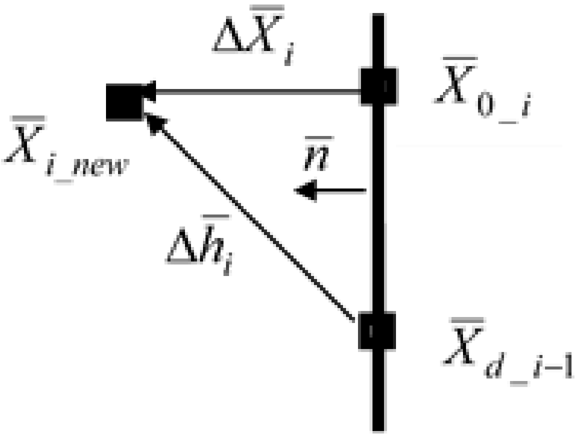


Fig. 2. Adjusting the coordinates of the mechatronic module's actuator.

For this purpose, new Cartesian coordinates [12,13] of the mechatronic module's actuator are determined:

$$\bar{X}_{i_{\text{new}}} = \bar{X}_{0_i} + \Delta\bar{X}_i,$$

where: $\Delta\bar{X}_i = \bar{n}\cdot k_i -$ increment of Cartesian coordinates of the tool position; $\bar{n} -$ unit normal to the surface (directed from the surface); $k_i = y(N_{0_i} - N_{d_{i-1}})$ – proportional coefficient (the greater the difference between the actual force at the previous step $N_{d_{i-1}}$ and the specified force at the i-th step N_{0_i} the greater the coefficient and the further the tool will move away from the surface); $y -$ correction coefficient depending on the physical parameters of the system (surface type, material, type of actuator, etc.); $\bar{X}_{0_i} -$ specified value of Cartesian coordinates at the *i*-th step [14,15]. The increments of the generalized coordinates $\Delta\bar{q}_i$, which allow the tool to be moved from point $(i-1)$ to point (i_{new}), are determined from the following system of equations:

$$\begin{cases}(\bar{X}_{i_{new}} - \bar{X}_{d_{i-1}} - J(\bar{q}_{0_i})\cdot\Delta\bar{q}_i)^T(\bar{X}_{i_{new}} - \bar{X}_{d_{i-1}} - J(\bar{q}_{0_i})\cdot \\ \left(\bar{M}_{0_i}\cdot\Delta\bar{q}_i\right)^T\left(\bar{M}_{0_i}\cdot\Delta\bar{q}_i\right)\to min,\end{cases}$$

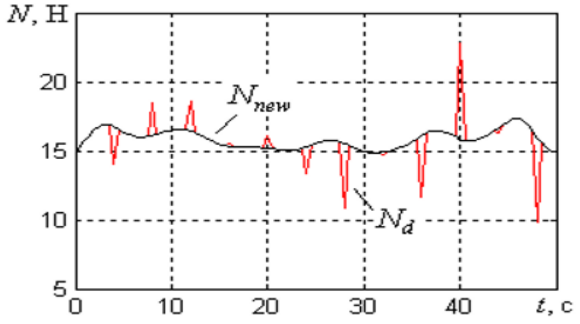
where: $J(\bar{q}_{0_i})$ is the Jacobian matrix; $J(\bar{q}_{0_i})\cdot\Delta\bar{q}_i = \Delta\bar{h}_i$; $\Delta\bar{h}_i$ is the displacement from point $\bar{X}_{d_{i-1}}$ to point $\bar{X}_{i_{\text{new}}}$; $\bar{M}_{0_i}$ is the vector of given generalized moments at the i-th step [11,16]. The first equation of this system ensures the displacement of the executive element of the mechatronic module to point (i_{new}), and the second one minimizes the work performed by the module when moving from point $(i-1)$ to point (i_{new}). To determine the optimal increments of the generalized coordinates $\Delta\bar{q}_i$, ensuring the maintenance of a given tool pressing force $\bar{N}_i^0$, it is proposed to use a vector criterion:

$$(A + B\cdot\Delta\bar{q}_i)^T(A + B\cdot\Delta\bar{q}_i)\to min, \qquad (5)$$

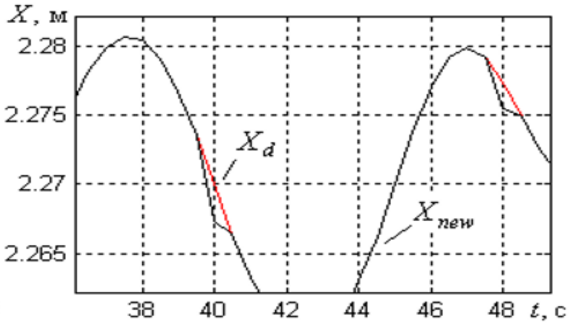
where

$$A = [\bar{X}_{i\_new} - \bar{X}_{d\_i-1} \quad 0]^T, B = [-J(\bar{q}_{0\_i}) \quad \bar{M}_{0\_i}]^T.$$

This criterion ensures an iterative search for increments in generalized coordinates taking into account the minimization of the work performed by the mechatronic module and should be used in module control algorithms when performing 3D graphics display operations [3,17]. Based on the proposed method of structural-parametric correction of pressing forces, a control algorithm was developed that ensures the correction of generalized coordinates in the event of exceeding the specified pressing forces. In order to confirm the operability of the control algorithm, a typical horizontal trajectory of the actuator of the mechatronic module, specified by Cartesian coordinates $\bar{X}_d$, was investigated. The required pressing forces on the surface varied within the range $10\,H \le N_d \le 20\,H$. During the algorithm operation, new coordinates of the trajectory of the actuator $\bar{X}_{\text{new}}$ were formed, ensuring the achievement of the permissible pressing forces $\bar{N}_{\text{new}}$. The results are presented in Fig. 3.



a)



b)

Fig. 3. Simulation results of the mechatronic module control algorithm:

a - change in the force exerted by the actuator on the surface;

b - change in the position of the actuator.

Simulation of the control algorithm showed that the proposed method of structural-parametric correction of the force exerted by the actuator on the surface of the mechatronic module reduces the time required to calculate control actions by using two-level control depending on the magnitude of the mismatch between the actual and specified force values.

CONCLUSION.

This article develops a comprehensive mathematical control model for a mechatronic module designed to accurately display 3D graphics on various surfaces according to specified coordinates and positions. The study systematically addressed the problems of transforming trajectory coordinates from the surface coordinate system to the module's base coordinate system, accounting for angular misalignments, and planning the actuator's motion in the presence of restricted and exclusion zones. The developed method enables the generation of straight-line and curved trajectories, taking into account surface geometry and dimensional constraints. The exclusion zone bypass algorithm is based on the criterion of minimizing the number of non-technological transitions and enables the determination of the optimal motion sequence through a tree-based analysis of trajectory sections. Furthermore, a two-level method for structural-parametric force correction is proposed to stabilize the force interaction between the actuator and the surface. The modeling results showed that the proposed control algorithms provide high accuracy, reduced computation time, and stable maintenance of specified actuation forces. Overall, the developed mathematical model takes into account the kinematic and dynamic characteristics of the mechatronic module and forms a theoretical and practical basis for the implementation of high-quality 3D visualization.

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